PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Sesquicentenary of Gauss's First Measurement of the Absolute Value of Magnetic Intensity

S. R. C. Malin

Phil. Trans. R. Soc. Lond. A 1982 **306**, 5-8
doi: 10.1098/rsta.1982.0060

Phil. Trans. R. Soc. Lond. A **306**, 5–8 (1982) [5]
Printed in Great Britain

Sesquicentenary of Gauss's first measurement of the absolute value of magnetic intensity

By S. R. C. Malin
Institute of Geological Sciences, West Mains Road, Edinburgh EH9 3LA, U.K.

The geomagnetic field and its changes with time are major tools for probing the Earth's deep interior. It is therefore particularly appropriate that the Royal Society Discussion Meeting on the Earth's core should coincide with the 150th anniversary of the first absolute measurement of the intensity of the geomagnetic field. This was only one of Carl Friedrich Gauss's many contributions, both direct and indirect, to geomagnetism. For example, it was Gauss who devised the methods of least squares and of spherical harmonic analysis. With Weber, he applied these to the geomagnetic field to show that it was nearly all of internal origin. Gauss was also the initiator of, and an active participant in, the 'Göttingen Magnetic Union', a scheme for the simultaneous observation of the magnetic field at widespread sites from which has developed the present worldwide network of magnetic observatories.

The particular achievement that we commemorate here was the determination by Gauss of the horizontal intensity of the geomagnetic field in units related to the millimetre, milligram and second. (This was one tenth of the unit that subsequently bore his name.) At that time he was working towards a universal system of units for all physical quantities and conceived the original idea that magnetic intensity can be measured in terms of mass, length and time.

The experiment itself was in two parts. In the first part, called the vibration experiment, the period of oscillation of a magnet suspended horizontally from a tortionless fibre was determined by timing a large number of oscillations (see figure 1). The period, T, is related to the magnetic moment of the magnet, M, and the horizontal intensity of the geomagnetic field, H, by

$$T = 2\pi(I/MH)^{\frac{1}{2}},$$

where I denotes the moment of inertia of the magnet and is calculated from the dimensions and mass of the magnet. Hence

$$MH = 4\pi^2 I T^{-2}.$$

This equation is strictly valid only for small oscillations, and Gauss took care to reduce his determinations of T to the values that they would attain for oscillations of infinitesimal amplitude.

There is nothing particularly original about the vibration experiment. The proportionality between the inverse square of the oscillation period and the geomagnetic intensity in the plane of the oscillations was well known, and had been exploited (e.g. by Humboldt) for comparing the magnetic intensity in different parts of the world. The problem with such comparisons is that they depend on the properties of the magnet remaining constant, or at least changing uniformly between successive measurements at the base station. They offer no means of distinguishing between geomagnetic secular change and changes in the moment of the magnet.

The second experiment, usually called the deflexion experiment, was newly devised by Gauss and was only possible in the light of his deduction that the magnetic intensity of a dipole decreases as the inverse cube of the distance. Using this result, or a modification of it that allowed for the finite length of the magnet, Gauss was able to find the ratio of M to H by noting the deflexion of a compass needle when the magnet used in the first experiment was placed at various distances to the east or west of the compass. The experiment is illustrated in figure 2: the magnetic intensity at O due to a magnet of moment M placed as shown is $2M(1+k/d^2)/d^3$, where k is a small constant that results from the finite length of the magnet. The ratio between

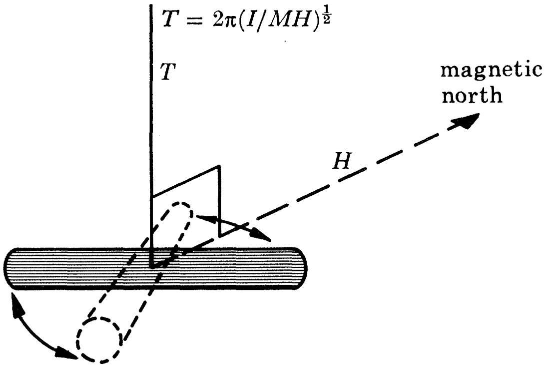

FIGURE 1. Experiment 1: the vibration experiment for the determination of MH.

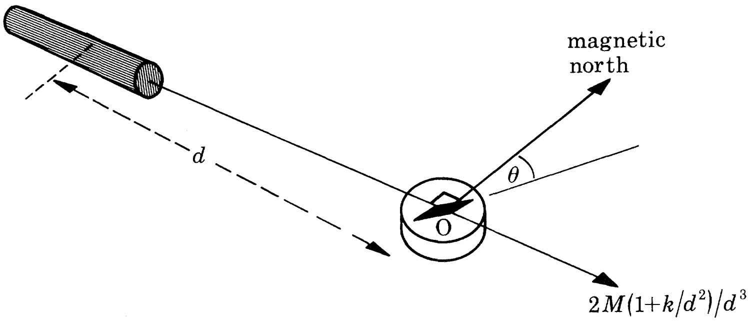

FIGURE 2. Experiment 2: the deflexion experiment for the determination of M/H.

this intensity and that of the Earth's magnetic field is tan θ, where θ is the deflexion from magnetic north of a compass needle at O. Hence

$$M/H = \tfrac{1}{2}d^3(1-k/d^2)\tan\theta.$$

The value of k was determined by least squares from observations of θ at different values of d.

Finally, H could readily be deduced by combining MH from the first experiment with M/H from the second.

The first such observation was made on 21 May 1832, and gave a value of 1.7820 $\mathrm{mg}^{\frac{1}{2}}$ $\mathrm{mm}^{-\frac{1}{2}}\,\mathrm{s}^{-1}$ (≡ 17820 nT) for the horizontal intensity in Göttingen. While it was certainly Gauss who calculated this value – his manuscript workings are preserved in the Göttingen University Library – it was almost equally certainly Wilhelm Weber who made the observations on which the calculations are based. The evidence for this is in a letter from Gauss to Olbers dated 18 February 1832, part of which U. Schmucker translates: 'I occupy myself now with the Earth's magnetism, particularly with an *absolute* determination of its intensity. Friend Weber conducts the experiments on my instructions.' This first observation was one of ten made between 21 May and 15 October 1832. They were published, together with a detailed description of the method in Latin, in the following year (Gauss 1833). The paper was subsequently republished in different journals in German, French and Italian, and it is probable that Gauss made the translation himself: he was an accomplished linguist and had at one time considered a career in philology rather than mathematics.

The apparatus required for Gauss's method was remarkably simple and portable. Though Gauss himself never travelled far from Göttingen, his method was soon being applied in many parts of the world, notably by Kreil, Hansteen and Sabine. In one of his letters to Herschel (written in perfect English) Gauss confides that he admires the work of Hansteen, but is not so sure about that of Kreil. For observatory work, the highest accuracy was desirable, and the apparatus was refined at the Kew observatory, Richmond, to attain this. The resulting Kew unifilar magnetometer was a beautiful instrument that permitted the absolute determination of declination as well as of horizontal intensity by using Gauss's method. From about 1860, this instrument came into almost universal use at magnetic observatories, and is still in use at some.

Gauss's method for the absolute measurement of magnetic intensity was not improved upon until the introduction of the coil magnetometer in the 1920s. This instrument was developed by F. E. Smith at the National Physical Laboratory, based on principles set out by Sir Arthur Schuster. If a known current is passed through a coil of known dimensions, the magnetic field it produces at the centre can be calculated. The current is increased until the artificial field just balances the horizontal component of the Earth's magnetic field, when a magnet suspended at the centre will swing round. The accuracy of the instrument depends on the precision with which the current and coil dimensions can be measured.

Nowadays, absolute measurements of the intensity of the Earth's magnetic field are made quickly and easily with a proton magnetometer. Protons precess around magnetic lines of force at a rate that depends on the strength of the magnetic field. The precession of randomly oriented protons cannot be detected, but if their spin axes are initially aligned by a strong magnetic field, when it is removed they will precess around the geomagnetic field in unison, producing a signal, which can readily be detected. A proton magnetometer can measure the magnetic field to $5/10^6$ and is an excellent survey instrument.

Gauss's contribution to geomagnetism was only a small part of his prodigious scientific achievement. He is best remembered as a pure mathematician: for his construction of the regular 17-sided polygon, his four separate proofs of the fundamental theorem of algebra (the first of which earned him his Ph.D. from the University of Helmstadt) and for his contributions to the development of complex number theory. As an applied mathematician he developed the method of least squares, defined the Gaussian, or normal, distribution of errors, derived and applied the method of spherical harmonic analysis. He also made important contributions to optics, astronomy, geodesy and (again with Weber) to telegraphy.

For his determination of the orbit of Ceres he was elected a Foreign Member of the Royal Society in 1804, and in 1838 he was awarded the Royal Society's Copley Medal. In commemorating the sesquicentenary of Gauss's absolute observation we should recognize that we are not so much paying homage to an early scientist as congratulating ourselves that such a luminary should have chosen to honour our subject with his attention.

This note is essentially a written version of an exhibit that was on display in the Royal Society's City of London Rooms throughout the meeting. The exhibit was prepared in collaboration with Professor U. Schmucker and with valuable suggestions from Mr D. R.

Barraclough, Dr F. J. Lowes and Professor S. K. Runcorn, F.R.S. Mr Barraclough was also heavily involved in the transportation and erection of the exhibit. The display material was prepared at the University of Newcastle under the supervision of Mr G. Boulton and at the Institute of Geological Sciences, Edinburgh. Exhibition items were lent by: the Institute of Geological Sciences, the Science Museum, the Royal Greenwich Observatory, the Littlemore Scientific Engineering Company, the library of the Royal Society and the Universitäts-Bibliothek, Göttingen. To all these people and institutions I am most grateful. This paper is published with the permission of the Director of the Institute of Geological Sciences (N.E.R.C.).

Reference (Malin)

Gauss, C. F. 1833 *Intensitas vis magneticae terrestris ad mensuram absolutam revocata.* Göttingen.